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Fading of Radio Waves Scattered by Dielectric Turbulence

RICHARD A. SILVERMAN

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1. Introduction

The solution of the problem of the scattering of radio waves by aggregates of discrete scatterers in random motion ('chaff', rainfall, etc.) is by now classical[1], [2]. The time structure or fading characteristics of the scattered radiation is caused by Doppler shifting of the incident radiation by the separate scatterers, and is thus simply related to the velocity distribution of the scatterers. In recent years, the model of discrete scatterers has been borrowed to explain the fading of radio waves scattered by dielectric turbulence. This has led to the models of Booker and Gordon[3], Rice[4], and others, in which the turbulence, at least for the purposes of fading calculations, is represented as an aggregate of 'scattering blobs', with velocities resulting from turbulent fluctuations and convection by the mean wind. Such models lead immediately to the linear relation between fading rate and carrier frequency which is characteristic of Doppler shifting. In this paper we show how this model is modified when the specifically turbulent inner structure of the random dielectric constant field is taken into account, in particular when the dielectric turbulence obeys the Kolmogoroff-Obukhoff structure law as described elsewhere[5]. It is found that the dependence of fading rate on carrier frequency differs from that given by the scattering blob models.

2. Relation between the scattered radiation and the dielectric noise*

Suppose an antenna A emits monochromatic electromagnetic radiation of wavelength λ, which is scattered into a receiving antenna A', at a distance D beyond the geometrical horizon from A, by the dielectric noise in the common volume V of the antenna beams. For simplicity, and without loss of generality, we assume that V is a cube centered at the origin O of a cartesian coordinate system which is fixed with respect to the earth. Let $\vec{n}$ be a unit vector along the line AO and $\vec{n}'$ a unit vector along the line OA'. Then the scattering wave vector $\vec{K}$ is the vector

$$\vec{K} = (2\pi/\lambda)\,(\vec{n}' - \vec{n}) .$$

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*In this section we assemble preliminary material for the considerations that follow; a more detailed discussion of this material with the same notation is given by the author[6].

In terms of the scattering angle θ (see Figure 1), we have*

$$K = |\vec{K}| = (4\pi/\lambda)\sin(\theta/2) \sim 2\pi\theta/\lambda = 2\pi\theta f/c ,$$

where f is the carrier frequency, and c is the velocity of light. (We neglect the variation of the antenna propagation factors in V, by setting them equal to unity everywhere in V.)

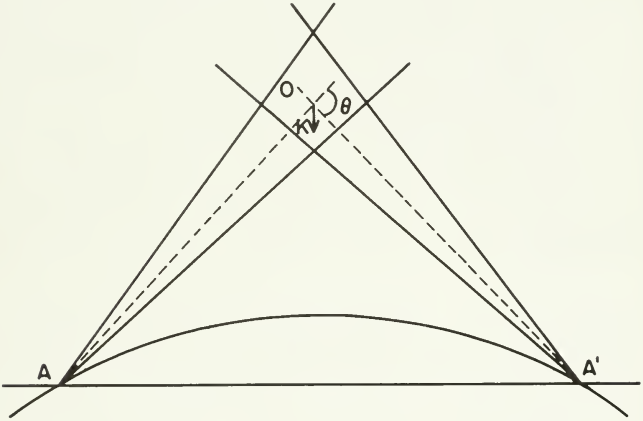

Figure 1

Geometrical configuration of a radio scattering experiment (schematic).

The received scattered radiation will be described by giving the statistical structure of the random function I(t), the complex amplitude of the current in the receiving antenna load measured in units of peak amplitude of the current that would appear in the receiving antenna load if the antennas were in free space a distance D apart and oriented with their directions of maximum gain along the line joining them. We recall the following facts about I(t):

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*We shall use the symbol ∼ to mean approximately equal to, sometimes only to within an order of magnitude.

1. To a good approximation, $I(t)$ is a complex gaussian random function of time, with mean zero. Accordingly, $I(t)$ is specified (at least to the extent that it is gaussian) by giving its covariance $\langle I(t)I^*(t+\tau)\rangle$. (The asterisk denotes the complex conjugate, and the angular parentheses mean the ensemble average.)

2. $I(t)$ is approximately stationary for time intervals of several minutes duration. Thus the covariance of $I(t)$ is substantially independent of t for minutes at a time, i.e.,

$$\langle I(t)I^*(t+\tau)\rangle \sim \langle |I|^2\rangle \; C(\tau) ,$$

where the correlation function $C(\tau)$ is a function only of the difference variable $\tau = t-t'$.

By the dielectric noise $\triangle\varepsilon(\vec{r},t)$ we mean the random departure from its mean of the relative dielectric constant at the point $(\vec{r},t)$. We make the following assumptions about $\triangle\varepsilon(\vec{r},t)$:

3. $\triangle\varepsilon(\vec{r},t)$ is approximately stationary in space within V and in time for intervals of the order of several minutes (as is $I(t)$); thus we can write the covariance of $\triangle\varepsilon(\vec{r},t)$ as

$$\langle \triangle\varepsilon(\vec{r},t)\,\triangle\varepsilon(\vec{r}\,',t')\rangle \sim \langle (\triangle\varepsilon)^2\rangle \; R(\vec{\rho},\tau) ,$$

where the correlation function $R(\vec{\rho},\tau)$ is a function only of the coordinate differences $\vec{\rho} = \vec{r}-\vec{r}\,'$ and $\tau = t-t'$.

4. For every fixed t, the random function $\triangle\varepsilon(\vec{r},t)$ is substantially auto-correlated only over a distance L_o, the macro-eddy dimension of turbulence theory*; it is difficult to define L_o to within better than an order of magnitude.

Two other assumptions that we shall make are:

5. The magnitude of the scattering vector satisfies the inequality $KL_o \gg 2\pi$. In qualitative language, the eddies of wave number $\vec{K}$, the so-called scattering eddies, are at least an order of magnitude smaller than the macro-eddies. This condition is needed if statistical turbulence theory is to be used to make deductions about $\triangle\varepsilon(\vec{r},t)$ and consequently about $I(t)$.

6. The underlying turbulent velocity field $\vec{v}(\vec{r},t)$, which convects the dielectric noise, has the same stationarity properties and correlation distance as

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*We assume some acquaintance on the part of the reader with the terminology and results of statistical turbulence theory. Basic references are G.K. Batchelor[7] and L. Landau and E. Liftshitz[8].

$\triangle\,\varepsilon(\vec{r},t)$. (Thus must be the case if the dielectric noise is caused by turbulent mixing[5].)

The space-time spectral density $\Psi(\vec{k},\omega)$ of the random process $\triangle\,\varepsilon(\vec{r},t)$ and the space-time correlation function $R(\vec{\rho},\tau)$ are related by the familiar Wiener-Khintchine formulas

$$\Psi(\vec{k},\omega) = (2\pi)^{-4} \int R(\vec{\rho},\tau) \exp\left[i(\vec{k}\cdot\vec{\rho}+\vec{\omega}\tau)\right] d\vec{\rho}\, d\tau \ ,$$

$$R(\vec{\rho},\tau) = \int \Psi(\vec{k},\omega) \exp\left[-i(\vec{k}\cdot\vec{\rho}+\vec{\omega}\tau)\right] d\vec{k}\, d\omega \ ,$$

where all variables of integration range from $-\infty$ to ∞. Note that $\Psi(\vec{k},\omega)$ is normalized, i.e., that

$$\int \Psi(\vec{k},\omega)\, d\vec{k}\, d\omega = 1 \ .$$

Of particular interest is the quantity $\Phi(\vec{k},\tau)$ defined as

$$(1) \quad \Phi(\vec{k},\tau) = \int \Psi(\vec{k},\omega) \exp(-i\omega\tau) d\omega = (2\pi)^{-3} \int R(\vec{\rho},\tau) \exp(i\vec{k}\cdot\vec{\rho})\, d\vec{\rho} \ .$$

The mixed quantity $\Phi(\vec{k},\tau)$ is a spectral density in the variable $\vec{k}$, and an unnormalized correlation function in the variable τ; we shall call $\Phi(\vec{k},\tau)$ the (spatial) semitransform of the random process $\triangle\,\varepsilon(\vec{r},t)$.

To a good approximation, the relation between the covariance of the received scattered current and the dielectric noise is given by the formula

$$(2) \quad \left\langle I(t) I^*(t+\tau) \right\rangle \sim (\pi D/\lambda^2 R R')^2 (2\pi)^3 V \left\langle (\triangle\,\varepsilon)^2 \right\rangle \Phi(\vec{K},\tau) \ ,$$

where $\vec{K}$ is the scattering wave vector, R is the distance from A to O, and R' is the distance from A' to O. Dividing equation (2) by $\left\langle |I|^2 \right\rangle$, we obtain

$$(3) \qquad C(\tau) \sim \Phi(\vec{K},\tau) / \Phi(\vec{K},0) \ .$$

Equation (3) is the starting point for our analysis of the time structure of $I(t)$.

3. Convection of the dielectric noise and its effect on the scattered radiation

As just defined, the dielectric noise $\triangle \varepsilon(\vec{r},t)$ and its semitransform $\oint(\vec{k},\tau)$ are measured in a coordinate system fixed with respect to the earth. On the other hand, turbulence theory leads most directly to statements about quantities measured in a Lagrangian coordinate system, i.e., a system moving with the local wind velocity* $\vec{v}(\vec{r},t)$. We must now separate the contribution to $\oint(\vec{k},\tau)$ due to convection by the velocity field from the contribution due to time variation of the microstructure of the dielectric turbulence as seen in the Lagrangian system. Things are considerably simplified if we realize from the outset that the convective effect is due primarily to the mean velocity field and the macro-eddies, and that the macro-eddy velocities change only slightly during the correlation time of the received scattered radiation, which experiment shows to be of the order of seconds at most (cf., e.g., Trolese [11]). Let us now divide the scattering volume V into n equal cubical subvolumes $V_1,\dots,V_n$ of side d satisfying the relation $L \ll d < L_0$, where** $L = 2\pi/K$. Then our fading calculations will be accurate to within at least an order of magnitude if we replace $\vec{v}(\vec{r},t)$ in each subvolume V_i by an average value $\vec{v}_i$, obtained by space-averaging $\vec{v}(\vec{r},t)$ over V_i and time averaging $\vec{v}(\vec{r},t)$ over an interval of the order of the correlation time of the received scattered radiation. It follows from the stationarity properties of $\vec{v}(\vec{r},t)$ that the random variables $\vec{v}_i$, $1 \leq i \leq n$, are identically distributed with probability density $p(\vec{v}) = p(v_x,v_y,v_z)$, i.e., the probability that any of the $\vec{v}_i$ has components lying in the intervals (v_x,v_x+dv_x), (v_y,v_y+dv_y), (v_z,v_z+dv_z) is given by $p(v_x,v_y,v_z)$. Moreover, $p(\vec{v})$ is approximately the univariate probability density of the velocity field $\vec{v}(\vec{r},t)$, despite the smoothing out of the finer fluctuations entailed in the definition of the $\vec{v}_i$.

We now represent $C(\tau)$ as a sum of contributions coming from the different V_i, i.e., we write***

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* See relevant remarks by G.K. Batchelor [7], p. 120; W. Heisenberg [9], Section 5; and A.N. Kolmogoroff [10].

**The condition that $L \ll d$ is required for the validity of equation (4) below; see [6]. In typical radio scattering experiments the relative size of V and L_0^3 is such that n will be a very large number.

***Since the volumes V_i are large compared to the size of the scattering eddies, there is negligible correlation between the scattering eddies in the different V_i. Thus in (4) we can neglect interactions between the contributions to I(t) coming from different V_i.

$$C(\tau) \sim (1/n) \sum_{i=1}^{n} \Phi_i(\vec{K},\tau)/\Phi_i(\vec{K},0) \quad . \tag{4}$$

The subscript on $\Phi_i(\vec{K},\tau)$ indicates that it pertains to the subvolume V_i, where the local wind velocity is approximately $\vec{v}_i$. $\Phi_i(\vec{K},\tau)$ is to be calculated in a coordinate system fixed with respect to the earth. To separate the contribution to $\Phi_i(\vec{k},\tau)$ due to convection from that due to the time variation of the turbulence itself, we introduce the quantity $\Phi_m(\vec{k},\tau)$, the semitransform of the dielectric noise measured in the moving Lagrangian system; $\Phi_m(\vec{k},\tau)$ can be deduced from turbulence theory, at least in principle, and will be assumed to be the same function in all the V_i. The relation between $\Phi_i(\vec{k},\tau)$ and $\Phi_m(\vec{k},\tau)$ can be deduced as follows. We note first that by equation (1), $\Phi_i(\vec{k},\tau)$ is the ρ-transform of the correlation function $R_i(\vec{\rho},\tau)$ measured in the fixed volume V_i, whereas $\Phi_m(\vec{k},\tau)$ is the $\vec{\rho}$-transform of the correlation function $R_m(\vec{\rho},\tau)$ measured in the Lagrangian system. Let S_i be a coordinate system moving with the velocity $\vec{v}_i$, and let the space and time coordinates measured in S_i be $\vec{r}_i$ and t_i, related to the coordinates $\vec{r}$ and t of the fixed system by

$$\begin{aligned} \vec{r}_i &= \vec{r} - \vec{v}_i t \ , \\ t_i &= t \qquad , \qquad 1 \le i \le n \ . \end{aligned} \tag{5}$$

The difference variables $\vec{\rho}_i = \vec{r}_i - \vec{r}_i^{\,\prime}$, $\tau_i = t_i - t_i^{\prime}$ and $\vec{\rho} = \vec{r} - \vec{r}^{\,\prime}$, $\tau = t - t^{\prime}$ transform in the same way. Now by definition

$$R_i(\vec{\rho},\tau) = R_m(\vec{\rho}_i,\tau_i) \quad . \tag{6}$$

If we multiply both sides of (6) by $(2\pi)^{-3}\exp(i\vec{k}\cdot\vec{\rho})$, integrate from $-\infty$ to ∞ in all the components of $\vec{\rho}$ and use (5), we find

$$\begin{aligned} \Phi_i(\vec{k},\tau) &= (2\pi)^{-3}\int R_i(\vec{\rho},\tau)\exp(i\vec{k}\cdot\vec{\rho})d\vec{\rho} = (2\pi)^{-3}\int R_m(\vec{\rho}_i,\tau_i)\exp(i\vec{k}\cdot\vec{\rho})d\vec{\rho} \\ &= (2\pi)^{-3}\int R_m(\vec{\rho}_i,\tau_i)\exp\left[i\vec{k}\cdot(\vec{\rho}_i + \vec{v}_i\tau)\right]d\vec{\rho}_i = \exp\left[i(\vec{k}\cdot\vec{v}_i)\tau\right]\Phi_m(\vec{k},\tau_i). \end{aligned}$$

Thus the desired relation between $\Phi_i(\vec{k},\tau)$ and $\Phi_m(\vec{k},\tau)$ is

$$\Phi_i(k,\tau) = \exp\left[i(\vec{k}\cdot\vec{v}_i)\tau\right]\Phi_m(\vec{k},\tau) \quad . \tag{7}$$

With the use of (7), equation (4) becomes

$$C(\tau) \sim \left[\Phi_m(\vec{K},\tau)/\Phi_m(\vec{K},0)\right](1/n)\sum_{i=1}^{n}\exp\left[i(\vec{K}\cdot\vec{v}_i)\tau\right] \ . \tag{8}$$

We assume that the random process $\vec{v}(\vec{r},t)$, and consequently the random variables $\vec{v}_i$, have the ergodicity needed to identify space and ensemble averages. Thus the last factor in (8) can be written

$$(9) \qquad (1/n) \sum_{i=1}^{n} \exp\left[i(\vec{K}\cdot\vec{v}_i)\tau\right] \sim \int \exp\left[i(\vec{K}\cdot\vec{v})\tau\right] p(\vec{v})\, d\vec{v} = \chi(\tau) ,$$

where $p(\vec{v})$ is the common probability density of the random variables $\vec{v}_i$. The integral in (9) is the Fourier transform of a non-negative function, and hence, by a theorem of Bochner-Khintchine*, it is a correlation function, which we have denoted by $\chi(\tau)$. Thus the correlation function of the received scattered signal is the product of two correlation functions

$$(10) \qquad C(\tau) \sim \left[\Phi_m(\vec{K},\tau)/\Phi_m(\vec{K},0)\right]\chi(\tau) \equiv \phi(\vec{K},\tau)\chi(\tau) ,$$

the first of which represents the fading due to time variation of the scattering eddies, and the second of which represents the fading due to convection of the scattering eddies by the mean wind and the macro-eddies. (The reader is reminded that the product of two correlation functions must be a correlation function (cf. Loève [12], p. 468).) If the scattering eddies are constant in time, so that $\phi(\vec{K},\tau) \equiv 1$, we have simply

$$C(\tau) \sim \chi(\tau) = \int \exp\left[i(\vec{K}\cdot\vec{v})\tau\right] p(\vec{v})\, d\vec{v} ,$$

which is recognized as the standard expression for the Doppler fading of signals returned by randomly moving aggregates of discrete scatterers [1], [2].

To derive expressions suitable for comparison with experiment, we pass from the complex random process $I(t)$ to its absolute value or envelope $|I(t)|$. Since $I(t)$ is a complex gaussian process, with identically distributed and orthogonal real and imaginary parts**, it follows that $|I(t)|$ is a Rayleigh process. By this we mean that the distribution of the random variables

$$|I(t_1)|, \ldots, |I(t_n)|$$

for any $t_1,\ldots,t_n$, and n, is the appropriate multivariate Rayleigh distribution given by Hoffman [14]. The envelope correlation function is the quantity

$$C_e(\tau) = \left\langle (|I(t)| - \langle |I| \rangle)(|I(t+\tau)| - \langle |I| \rangle) \right\rangle .$$

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*See M. Loève [12], pp. 207, 482 and B.V. Gnedenko [13].

**These properties of the real and imaginary parts of $I(t)$ are a consequence of the reality and stationarity of $\Delta\varepsilon(\vec{r},t)$.

It is easily shown[15] that the relation between $C_e(\tau)$ and $C(\tau)$ is

(11) $$C_e(\tau) = \frac{\pi}{4-\pi}\left[\frac{1}{4}C^2(\tau) + \frac{1}{64}C^4(\tau) + \dots\right] .$$

We now choose as a convenient measure of the envelope fading rate the quantity Z, the expected number of zero crossings per second of the centered random process $|I(t)| - \langle |I| \rangle$. By a well-known formula[16]

$$Z = (1/\pi)\sqrt{-C_e''(0)} ,$$

where the prime denotes differentiation with respect to τ. By (10) and (11)

(12) $$Z \sim \gamma\sqrt{-\phi''(\vec{K},0) - \chi''(0)} , \qquad \gamma = \left[(2\pi)(4-\pi)\right]^{-1/2} ,$$

where we have used

$$\phi(\vec{K},0) = \chi(0) = 1 \quad \text{(normalization)},$$

$$\phi'(\vec{K},0) = \chi'(0) = 0 \quad \text{(oddness)} .$$

To evaluate $\chi''(0)$, we expand the exponential in the defining integral (9) and find that

(13) $$-\chi''(0) = \int (\vec{K}\cdot\vec{v})^2\, p(\vec{v})\, d\vec{v} \sim \langle(\vec{K}\cdot\vec{v})^2\rangle .$$

In radio scattering experiments $\vec{K}$ is usually directed vertically downward (see Figure 1). Thus, combining (12) and (13), we have finally

(14) $$Z \sim \gamma\sqrt{-\phi''(\vec{K},0) + \langle v_z^2\rangle K^2} .$$

If the scattering eddies do not vary in time, equation (14) reduces to

(15) $$Z \sim \gamma\langle v_z^2\rangle^{1/2} K \sim (2\pi\theta\gamma/c)\langle v_z^2\rangle^{1/2} f ,$$

the familiar linear Z-versus-f law characteristic of Doppler shifting. Moreover, if the time variation of the scattering eddies is such that $\phi''(\vec{K},0)$ is proportional to K^2, i.e.,

$$-\phi''(\vec{K},0) = a K^2 ,$$

then (14) becomes

$$(16) \qquad Z \sim (2\pi\theta\gamma/c)\sqrt{a + \langle v_z^2 \rangle}\; f \;.$$

It is important to note that (15) and (16) are not only the equations of straight lines, but of straight lines passing through the origin of the (Z,f) coordinate system. Equations (15) and (16) correspond to the results of earlier work [3], [4], where the dielectric turbulence itself was represented heuristically as a cloud of independent discrete scatterers. As we shall see below, the turbulent mixing model of dielectric turbulence leads to a different K-dependence for $\phi''(\vec{K},0)$ and to a different Z-versus-f law.

4. Fading rate with turbulent mixing model of dielectric turbulence

To apply statistical turbulence theory to an analysis of $\phi''(\vec{K},0)$, we shall assume that the dielectric noise is produced by turbulent mixing of dielectric constant inhomogeneities*. Guided by the work of Obukhoff [17] and Corrsin [18], we assume that the turbulently mixed field $\triangle\, \varepsilon(\vec{r},t)$ has the same microstructure as the underlying turbulent velocity field $\vec{v}(\vec{r},t)$. With such a model we are justified in inferring the time structure of dielectric constant eddies of a given size from the time structure of velocity eddies of the same size. On the other hand, if dielectric turbulence were due to some other mechanism, e.g., the density fluctuations associated with turbulent pressure fluctuations, we could not make this identification. Indeed, in the case of the pressure fluctuation model, now abandoned as an important source of dielectric turbulence, the time structure of pressure eddies of a given size is related in a complicated way to the time structure of velocity eddies of many different sizes [15].

Let $\Phi_v(\vec{k},\tau)$ be the semitransform of the turbulent velocity field defined by (1), with $\Psi(\vec{k},\omega)$ and $R(\vec{\rho},\tau)$ interpreted as the spectral density and correlation function of the velocity field. Despite the fact that a satisfactory derivation of $\Phi_v(\vec{k},\tau)$ has not yet been achieved (except for $\tau = 0$), we can infer the quantity $\Phi''_v(\vec{k},0)$, and presumably $\phi''(\vec{k},0)$, to within a constant of proportionality, by using dimensionality and similarity considerations. The argument is familiar from the work of von Weizsäcker [19] and Heisenberg [9], and goes as follows. If the Reynolds number of the turbulent flow is large, the rate at which kinetic energy is transferred from eddies larger than $\ell = 2\pi/k$ to eddies smaller than ℓ is independent of ℓ, provided that ℓ lies in the *inertial range*[7], [8]. Thus, if v_ℓ is the velocity of eddies of size ℓ, we have

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*The microstructure of dielectric turbulence is still a very open issue. Thus, the considerations of this section are perforce of a speculative nature. Evidence supporting the view that dielectric turbulence is due primarily to turbulent mixing of dynamically passive quantities like temperature, humidity, and electron density, is presented in [5].

$$v_\ell^3/\ell = \text{const.} \tag{17}$$

Defining a characteristic time $\tau_\ell = \ell/v_\ell$ for the eddies of size ℓ, and using (17), we find

$$\tau_\ell = \text{const.}\ \ell^{2/3} .$$

We now assume, following Heisenberg, that $\Phi_v(\vec{k},\ell)$ is a *universal* function of the dimensionless variable τ/τ_ℓ, which is proportional to $k^{2/3}\tau$. (The fact that $\Phi_v(\vec{k},\tau)$ does not depend on the direction of $\vec{k}$ is a consequence of the local isotropy of eddies in the inertial range[7], [8].) Thus, assuming that the time structure of the dielectric constant eddies 'parallels' that of the velocity eddies, we can write

$$\phi(\vec{k},\tau) = g(k^{2/3}\tau) .$$

The desired expression for $\phi''(\vec{K},0)$ is therefore

$$\phi''(\vec{K},0) = g''(0)\,K^{4/3} . \tag{18}$$

Since correlation functions are concave downward, we know that $g''(0)$ is negative, but the exact value of $g''(0)$ cannot be determined without a more quantitative theory.

If we substitute (18) in (14), we obtain

$$Z \sim \gamma\sqrt{-g''(0)\,K^{4/3} + \langle v_z^2\rangle K^2} . \tag{19}$$

Defining a wave number K_0 by

$$K_0 = \left[-g''(0)/\langle v_z^2\rangle\right]^{3/2} ,$$

we can write (19) as

$$Z \sim \gamma\langle v_z^2\rangle^{1/2}\,K_0\sqrt{(K/K_0)^{4/3} + (K/K_0)^2} . \tag{20}$$

The validity of (20) depends on two conditions, which are equivalent to the requirement that the scattering eddies lie in the inertial range. The first is that $L = 2\pi/K \sim \lambda/\theta$ satisfy the inequality $L \ll L_o$, i.e., that the size of the scattering eddies be much less than the macro-eddy dimension. This condition seems to be met in most radio scattering experiments, both in the ionosphere and in the troposphere. The second condition is that $L \gg \eta$, where η is the characteristic dimension of the smallest eddies, which is determined by the action of molecular dissipation processes. This condition does not seem to be met in most ionospheric scattering experiments*. Thus, although the results of previous sections apply to both ionospheric and tropospheric scattering, the application of equations (17) through (20) is limited to tropospheric experiments, where the condition $L \gg \eta$ seems always to be satisfied.**

An alternate form of (20) in terms of the carrier frequency is

$$Z \sim \left(2\pi\theta\gamma \left\langle v_z^2\right\rangle^{1/2}/c\right) f_o \sqrt{(f/f_o)^{4/3} + (f/f_o)^2} \,, \tag{12}$$

where now

$$f_o = (c/2\pi\theta)\, K_o \,.$$

In Figure 2 we plot the function $h(x) = \sqrt{x^{4/3} + x^2}$, which is the Z-versus-f law if Z is measured in units of $(2\pi\theta\gamma \left\langle v_z^2\right\rangle^{1/2}/c) f_o$ and f is measured in units of f_o. Noticeable departures from the Doppler laws given by equations (15) and (16) can occur in two ways. First of all, if (21) is valid at frequencies $f \lesssim f_o/2$, which it may not be***, then at such frequencies the Z-versus-f law is nonlinear (see part of graph near origin). On the other hand, even if (21) does not apply at frequencies $f \lesssim f_o/2$, so that (21) is valid only for (Z,f) points lying on the portion of h(x) which is closely approximated by a straight line, the continuation of this straight line will not pass through the origin, as predicted by the Doppler laws (15) and (16) (see broken line in Figure 2). Thus, in any case, time variation of the scattering eddies should produce a measurable effect on the time structure of radio waves scattered by dielectric turbulence in the troposphere.

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* See, for example, F. Villars and V.F. Weisskopf [20] and A.D. Wheelon [21].

** For estimates of η in the troposphere, see A.M. Obukhoff [17] and G.K. Batchelor [22].

*** Corresponding to the condition $L \ll L_o$ or $KL_o \gg 2\pi$, equation (21) is valid only for frequencies such that $\theta f L_o/c \gg 1$. Thus, if a noticeable departure from linearity is to be predicted by (21), we must have $\theta f_o L_o/c \gg 1$. Whether or not this condition is met, a crude estimate shows that f_o is a frequency of the order of megacycles, (i.e., comparable to radio scattering frequencies) and that the fading rate Z is a frequency of the order of several cycles per second (i.e., comparable to observed fading rates).

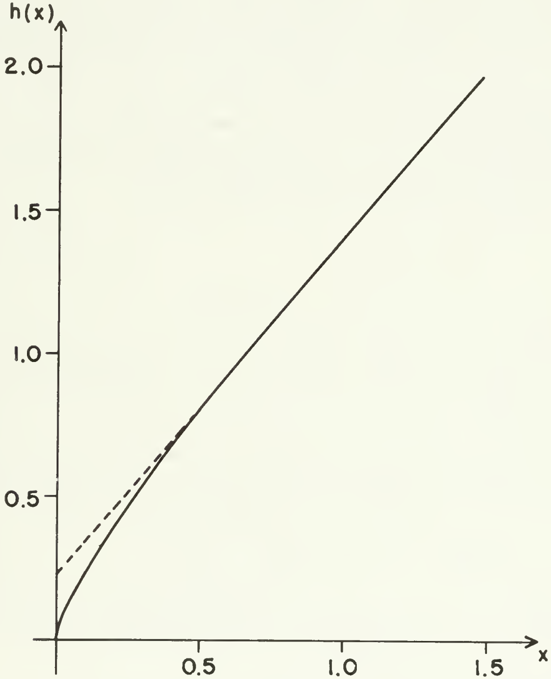

Figure 2

Graph of the function $h(x) = \sqrt{x^{4/3} + x^2}$.

5. Conclusions

We have shown that the time structure of electromagnetic radiation scattered by a continuous dielectric constant field contains contributions from two sources. One is the time variation of the spectral density of the random process at the scattering wave number, as seen in a system moving with the local wind velocity, which in the case of atmospheric turbulence is itself a random process. The other is Doppler shifting due to convection of the dielectric constant patterns that cause scattering, the so-called scattering eddies, by the mean wind and by the large velocity eddies. Our result is summarized in equation (10), which represents the correlation function of the received scattered current as the product of two correlation functions, one representing time variation of the scattered eddies, and the other convective Doppler shifting.

On less certain ground, we have assumed that dielectric turbulence is produced by the turbulent mixing of dielectric constant inhomogeneities, and obeys the Kolmogoroff-Obukhoff structure law. Although a complete solution of the space-time structure of turbulence, in particular of dielectric turbulence, is not yet available, it is possible, by the use of familiar dimensionality and similarity arguments, to deduce the dependence of the fading rate of the received envelope on the carrier frequency. The result is summarized in equation (21) and Figure 2, which apply only to tropospheric scattering and which exhibit significant departures from the Doppler law that goes with the scattering blob models. Experimental evidence in favor of the fading rate law of (21), as opposed to the Doppler law, appears indecisive. For example, Trolese[11] reports simultaneous measurements of the fading rate (defined somewhat differently) at three frequencies, the minimum number of points needed to decide for or against a linear law. He notes that of his two sets of data, consisting of three (Z,f) points each, only one set fits fairly well on a straight line through the origin. It is clear that a decisive test of (21) awaits more extensive experiments, which it is hoped this paper may in some measure stimulate.

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Bldg. 12, Rm. 2529

Smyth Research Associates
3930 4th Avenue
San Diego 3, California
ATTN: Dr. John B. Smyth

Electrical Engineering
California Institute of Tech.
Pasadena, California
ATTN: Dr. Georges G. Weill

Naval Research Laboratory
Washington 25, D.C.
ATTN: Dr. Henry J. Passerini
Code 5278 A

Dr. George Kear
10585 N. Stelling Rd.
Cupertino, California

Brooklyn Polytechnic Institute
85 Livingston Street
Brooklyn, New York
ATTN: Dr. Nathan Marcuvitz

Department of Electrical Engr.
Brooklyn Polytechnic Institute
85 Livingston Street
Brooklyn, New York
ATTN: Dr. Jerry Shmoys

Dept. of Mathematics
University of New Mexico
Albuquerque, New Mexico
ATTN: Dr. I. Kolodner

W.L. Maxon
460 W. 34th Street
New York, N.Y.
ATTN: Dr. Harry Hochstadt

NEW YORK UNIVERSITY

Institute of Mathematical Sciences
Division of Electromagnetic Research

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FADING OF RADIO WAVES SCATTERED BY DIELECTRIC TURBULENCE

Richard A. Silverman

Richard A. Silverman
Richard A. Silverman

Morris Kline
Morris Kline
Project Director

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Abstract

The fading of radio waves scattered by dielectric turbulence is shown to be the result of two effects. The first is time variation of the scattering eddies as seen in a coordinate system moving with the local wind velocity. The second is Doppler shifting produced by the convection of the scattering eddies by the mean wind and by the macro-eddies. In the troposphere, the scattering eddies lie in the inertial range of statistical turbulence theory. This makes it possible to find the envelope fading rate of the received scattered signal to within a constant of proportionality by using dimensionality and similarity arguments. The result is an expression for the fading rate as a function of carrier frequency which departs significantly from that found with 'scattering blob' models.

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